STATISTICAL ESTIMATION OF SEMANTIC PROVABILITY

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1. Introduction

Let us point out that there is nothing unexpected in this paper. The sole element of novelty is the formal description of a simple relation between a chapter of mathematical logic and mathematical statistics. The word semantic occurring in the title indicates that, roughly speaking, provability or nonprovability is to be estimated on the basis of truth and falsehood in interpretations in models. The logical formalism used in this paper is monadic logic introduced by P. R. Halmos in [2]. In principle it is possible to replace the monadic logic by a more developed formalism, for instance, by polyadic logic [3]. The elements, the provability or nonprovability of which is to be estimated, as well as the interpretations, are chosen at random by appropriate chance mechanisms, hence the whole problem is probabilistic in nature. The estimation procedures established in this paper possess a natural optimum property. The study of the behavior of these procedures at infinity shows that the statistical decision functions of finite size, which estimate provability are, in fact, asymptotically good approximate proofs. One may hope that the questions treated in this paper reflect at least the most elementary features of heuristic reasoning which is so perfectly realized by the human brain.

All that is necessary for an easy understanding is developed in the paper in full detail and with intuitive justification. The main reason is that one cannot expect that, in general, specialists in mathematical logic are familiar with concepts, methods and results of statistical decision theory or that statisticians are familiar with formalisms of mathematical logic.

The basic concepts and results of statistical decision theory on an appropriate level of generality are summarized in section 2. These results are then applied in section 3 to the problem of statistical estimation of belonging relations. The passage from the considerations of section 3 to the solution of our main problem of statistical estimation of provability is completely transparent and forms the contents of section 4.

The present paper, which is closely connected with [8], does not furnish more than may be intuitively expected and, therefore, its practical value is very limited. Further developments in this direction, however, will probably throw some light into the mechanism of human behavior in problem solving.