THE ESTIMATION OF DISCONTINUITIES IN MULTIVARIATE DENSITIES, AND RELATED PROBLEMS IN STOCHASTIC PROCESSES

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1. Summary

We consider estimation of the finite dimensional real parameter α from independent identically distributed observations on a generalized random variable with density discontinuous in the parameter on a set which depends on the parameter. We assume that certain regularity conditions are met; some of these will be explicitly stated, but many will not. These conditions are merely to insure that the approximations made are valid. Subject to these conditions, we show that hyper-efficient estimators, among them the maximum likelihood estimator, exist, and that asymptotically the estimation problem is equivalent to that for a nonstationary process with possibly multidimensional "time" and unknown center of nonstationarity.

The inference problem for the process is treated from the standpoint of the likelihood function, a method which has not been used as much as it should be.

2. Reduction to the asymptotic problems

Specifically, we consider the following problem: Let $x = (y, z)$, where y is k-dimensional, and let $R(\alpha)$ be a region in k-dimensional Euclidean space. We assume that the density is given by

$$f(x, \theta, \alpha) = g(x, \theta, \alpha), \qquad y \in R(\alpha), \tag{1}$$

$$f(x, \theta, \alpha) = h(x, \theta, \alpha), \qquad y \notin R(\alpha). \tag{2}$$

We do not assume that θ or z are finite dimensional. This density is with respect to $dy \times d\mu(z)$. From now on we shall assume all approcimations are valid.

One classic example is the case of the rectangular distribution with one or both endpoints unknown. Another is that of the endpoint of the exponential distribution. In both of these cases it is well known that the extreme order statistic or statistics are the maximum likelihood estimators and are hyperefficient.

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