

REGRESSION ANALYSIS OF CONTINUOUS PARAMETER TIME SERIES

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1. Summary

This paper attempts to show how the problem of regression analysis of time series can be treated, using Hilbert space techniques, in a manner which applies simultaneously to discrete and continuous parameter time series, and also to multiple time series. The idea of a Hilbert space representation of a time series and in particular the reproducing kernel Hilbert space representation, is discussed in section 3. Examples of such representations are given in section 4. A formula for the probability density functional of a normal time series is obtained in section 5. The problem of maximum likelihood estimation of the mean value function of a normal time series is treated in section 6. Minimum variance unbiased linear estimation of the mean value function is treated in section 7. Tests of hypotheses and simultaneous confidence bands for mean value functions are given in section 8. A method of iteratively evaluating reproducing kernel inner products is described in section 9.

2. Introduction

The problem of regression analysis of time series may be formulated as follows. A model often adopted for the analysis of an observed time series $X(t)$, $t \in T$, is to regard $X(t)$ as the sum of two functions

$$(2.1) \quad X(t) = m(t) + Y(t), \quad t \in T.$$

We call $m(t)$ the mean value function and $Y(t)$ the fluctuation function.

The stochastic process $Y(t)$ is assumed to possess finite second moments, and to have zero means and covariance kernel

$$(2.2) \quad K(s, t) = E[Y(s)Y(t)].$$

The mean value function is assumed to belong to a known class M of functions. Very often M is taken to be the set of all linear combinations of q known functions $w_1(t), \dots, w_q(t)$. Then, for t in T ,

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