OPTIMUM EXPERIMENTAL DESIGNS V, WITH APPLICATIONS TO SYSTEMATIC AND ROTATABLE DESIGNS

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1. Introduction and summary

In this paper we continue to develop the theory of construction of optimum experimental designs along the lines of [15], [19], [16], and [17]. Section 2 of the paper considers further general developments in both the exact and approximate theories, while in section 3 we apply the theory to construct optimum designs in the settings where systematic designs (subsection 3.1) and rotatable designs (subsection 3.2) are often employed, and in the setting of linear regression on an arbitrary Euclidean subset (subsection 3.3). Open problems are mentioned throughout the paper.

2. Generalities

2.1. *Notation and preliminaries.* Throughout this paper we shall achieve brevity by considering mainly a linear model. Corresponding asymptotic results in nonlinear problems hold and are obtainable without serious difficulty. One example of such a problem will be found in subsection 3.1 (ρ unknown), and further examples of explicit computations in certain nonlinear problems will be found in Chernoff [6] and Box and Lucas [4], while complete classes of designs for such problems were treated by the author (see [16], pp. 290–291). We shall also be primarily concerned with nonsequential designs, although one sequential problem is treated below theorem 3.1.2. The main idea in the construction of many such asymptotic sequential designs goes back to Wald [24], while recent work can be found in the papers of Chernoff [7] and his students.

We assume, then that $f_1, f_2, \cdots, f_k$ are k given real functions on a space $\mathfrak{X}$. Write f for the column vector of functions f_i. Let θ denote a real unknown column k-vector. Corresponding to each x in $\mathfrak{X}$, there is a random variable Y_x for which

$$E_\theta Y_x = \theta' f(x). \tag{2.1.1}$$

(Throughout this paper transposes are denoted by primes and subscripts on E

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