HYPOTHESIS TESTING AND ESTIMATION FOR LAPLACIAN FUNCTIONS

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1. Introduction

During the last few years, several papers have been devoted to the study of random functions. And though a large amount of work remains to be done in this field, it presents some difficulties. If we try to apply in the field of random functions—or, more generally, of random elements—some of the basic notions of classical mathematical statistics such as sufficient statistics or maximum likelihood, we find that conditional probabilities or probability densities do not obviously exist, that sets and spaces are not compact or even locally compact, and so on. The existence of conditional probabilities is a particularly important point. I emphasize that, in this paper, by "conditional probability" I always mean a "regular conditional probability," that is, with the *complete additivity* property.

Concerning this existence of conditional probabilities, a very important advance has been made by M. Jiřina [4]. Among the more general results given in this paper, there is the following statement.

THEOREM 1.1. *Let $\mathfrak{X}$ be a metric, separable, complete space of elements x, let S be the smallest σ-algebra of subsets of $\mathfrak{X}$ containing the spheres (or the Borel sets) of $\mathfrak{X}$, let $m(e)$ be a probability measure on $(\mathfrak{X}, S)$, that is to say, a function of the set e, defined for $e \in S$, which is nonnegative and completely additive on S, with $m(\mathfrak{X}) = 1$, and let Σ be any σ-algebra $\subset S$. Then there exists, associated with $m(e)$, at least one conditional probability $\mu(x; e)$ on S, relative to Σ, having the following properties.*

(a) *It is a nonnegative function of $x \in \mathfrak{X}$ and of $e \in S$, which, for every fixed $e \in S$, is Σ-measurable as a function of x.*

(b) *For every fixed x, it is a probability measure on S as a function of e, including complete additivity and $\mu(x; \mathfrak{X}) = 1$.*

(c) *For every $A \in \Sigma$ and every $e \in S$,*

$$m(A \cap e) = \int_A \mu(x; e) m\,(dx). \tag{1.1}$$

Jiřina has completed the preceding results in [5]. It must be pointed out that