

# MATHEMATICAL PROBLEMS IN THE SHANNON THEORY OF OPTIMAL CODING OF INFORMATION

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## 1. Introduction

1.1. The term “information theory” which has become fashionable during the last decade has as yet no unique interpretation. In technical and cybernetics literature, information theory is usually taken to mean the totality of the applications of mathematical methods to the problem of input, processing, storage, and transmission of information. From the mathematical point of view these applications have little in common since they are based on methods belonging to very diverse branches of mathematics. Such an interpretation of information theory covers the whole of mathematical statistics, much of the theory of random processes, the recently developed investigations into the power of the  $\epsilon$ -net in functional spaces [54], which is regarded as an estimate of the quantity of information given by an element of this space, estimates of the “complexity” of the algorithms of mathematical analysis [2] and [96], and so on.

But within information theory, in the wide sense of this term, an important place is occupied by a young discipline which is also (particularly in mathematical literature) often called information theory. To be explicit, we shall call it the Shannon theory of optimal coding of information. The reason is that everything in this discipline is the direct development of ideas contained in the remarkable and fundamental work of C. E. Shannon [78], and that the Shannon theory investigates means of transmitting information based on an optimal choice of methods of coding and decoding information. Moreover, the special characteristic of this theory is the possibility of greatly varying the method of coding information. In those cases where the coding method is rigidly fixed, the Shannon theory is not best suited to the problem, but rather the more usual methods of contemporary mathematical statistics should be employed. This is the case, for example, in the majority of statistical problems, when it is not within the power of the statistician to alter the procedure by which the relevant information was selected, and the only question which arises is the choice of an optimal decoding method (“the decision rule”).

1.2. It has recently become clear that the Shannon theory of coding information may be of interest to mathematicians not only because of the importance