ON THE SOLUTION OF TWO-STAGE LINEAR PROGRAMS UNDER UNCERTAINTY

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1. Introduction

In this paper we shall study the program whose constraints are given by

$$Ax + By = b, \qquad x \geqq 0, \qquad y \geqq 0 \tag{1}$$

where A and B are known $m \times n_1$ and $m \times n_2$ matrices, x and y are n_1 and n_2-dimensional vectors, and b is a random m-dimensional vector with known distribution. We wish to minimize with respect to x

$$E \min_y (c'x + f'y), \tag{2}$$

where c and f are known n_1 and n_2-dimensional vectors, E denotes expectation taken with respect to the distribution of b, and prime denotes transpose.

As an example of a situation giving rise to such a program, consider the set of possible polyhedra given by $Ax = b$, where $x \geqq 0$, when b is random. Here, in contrast to the usual case [6] where one minimizes $c'x$ subject to x lying in the intersection over b of these polyhedra, one is instead allowed, after selecting an x and subsequently observing b, to compensate with a vector $y \geqq 0$ for infeasibility of the selected x at a penalty cost $f'y$, where $f \geqq 0$. In this case By would be $y^+ - y^-$ and the vector y which yields the smallest penalty cost for each b and x would be composed of two parts, $y^+ = b - Ax$, $y^- = 0$ if $b \geqq Ax$ or $y^- = Ax - b$, $y^+ = 0$ if $b < Ax$. As choice of y depends on b as well as x, we alter the objective from minimizing $c'x$ to minimizing $c'x$ plus the expected smallest penalty cost.

Many short range inventory problems can be expressed mathematically as such a program. The vector x may represent an inventory which is to be bought at cost $c'x$ before the random demand b is observed. Once b is observed, one must compensate by a vector y, at cost f'y, for imbalances $(b - Ax)$ between the original inventory and the demand so as to satisfy (1). For example, coordinates of this vector y may represent the amount of additional inventory to be bought immediately to meet the excess of demand over supply or the amount of inventory to discard in case of an excess of inventory over demand.