ON PLANE SAMPLING AND RELATED GEOMETRICAL PROBLEMS

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1. Summary

We study the following problem. An isotropic plane stochastic process is observed at points making up a regular pattern. We are interested in finding patterns yielding the least limiting variance of the observed values, when the points are situated within a circle with infinitely increasing radius.

In section 4, a solution is presented for the correlation function (3.5) and some ranges of point densities. The solution is obtained by solving the related geometrical problem of covering a plane by circles in such a way that the circles mutually intersect as little as possible (see section 3). From the results obtained it follows that, in contradistinction to the linear case, no unique pattern of points is optimum for all convex correlation functions simultaneously. The efficiency of patterns in general use is, however, quite good.

In section 5, finally, we study a subclass of convex correlation functions of an isotropic plane process, consisting of functions that admit a spectral representation in terms of the simple correlation function (5.2).

2. Introduction

In this section, an expository survey of the background of the problem will be presented.

2.1. *Applications of plane sampling.* Many applications of the sampling method may be broadly described as "plane sampling." We give some examples.

Forest surveys. In the simplest case, one might want to estimate the area of a certain country or geographical district covered by forest. Similarly, one might want to estimate the proportion of a forest area covered by a certain variety of