TESTS OF SEPARATE FAMILIES OF HYPOTHESES

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1. Introduction

The aims of this paper are

(i) to stress the existence of a class of problems that have not received much attention in the literature;

(ii) to outline a general method, based on the Neyman-Pearson likelihood ratio, for tackling these problems;

(iii) to apply the general results to a few special cases.

Discussion of regularity conditions will not be attempted.

2. Some problems

The following are examples of the type of problem to be investigated. Throughout, Greek letters denote unknown parameters. It is assumed in the theoretical analysis that observed values of random variables are to be used to test one of the hypotheses, say H_f, and that high sensitivity is required for alternative hypotheses, H_g.

Example 1. Let $Y_1, \cdots, Y_n$ be independent identically distributed random variables. Let $H_f^{(1)}$ be the hypothesis that their distribution function is log-normal with unknown parameter values, and let $H_g^{(1)}$ be the hypothesis that their distribution function is exponential with unknown parameter value. For remarks on the difficulty of distinguishing these distributions, see [11]. A. D. Roy [20] has given a likelihood ratio test for the similar example of discriminating between a normal and a log-normal distribution. Other related examples include that of testing a Weibull-type distribution against a gamma distribution, and of testing alternative forms of quantal dose-response curves.

Example 2. Let $Y_1, \cdots, Y_n$ be independently normally distributed with constant variance and let $x_1, \cdots, x_n$ be given positive constants. Let $H_f^{(2)}$ be the hypothesis that

$$E(Y_i) = \alpha_1 + \alpha_2 x_i, \qquad i = 1, \cdots, n, \tag{1}$$

and let $H_g^{(2)}$ be the hypothesis that

$$E(Y_i) = \beta_1 + \beta_2 \log x_i, \qquad i = 1, \cdots, n. \tag{2}$$

Example 3. Let $\mathbf{Y}$ denote a vector of n independently normally distributed