SEQUENTIAL TESTS FOR THE MEAN OF A NORMAL DISTRIBUTION

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1. Summary

The problem of sequentially testing whether the drift of a Wiener process is positive or negative, given an a priori normal distribution, is reduced to the solution of a free boundary problem involving a diffusion equation.

2. Introduction

This paper is concerned with an approach to obtaining an asymptotically optimal solution (as sampling cost approaches zero) of the problem of sequentially testing whether the unknown mean μ of a normal distribution with known variance is positive or negative. That is the sequential test of $H_1\colon \mu > 0$ versus $H_2\colon \mu \leqq 0$.

In a number of problems of varying degrees of generality the following procedure has been found to yield asymptotically optimal solutions [6], [7]. This procedure consists of selecting some nondegenerate a priori distribution on the unknown states of nature and of studying the limiting behavior of the corresponding Bayes solution. Therefore we shall investigate the sequential problem where the a priori distribution of the unknown parameter will be assumed to be normal with fixed mean μ_0 and variance σ_0^2.

The nature of the Bayes solution will depend in part on the loss function. The case where there is an indifference zone, that is, the regret associated with either terminal action is zero in some interval about $\mu = 0$, has been solved and generalized by G. Schwarz [13]. A case which seems of more interest is that where the loss functions corresponding to the two terminal actions are approximately linear in a neighborhood of the origin. Then, the regret due to taking the wrong action may be expressed by

$$r(\mu) = k|\mu| + o(1) \tag{2.1}$$

as $\mu \rightarrow 0$. We shall confine our attention to this case although our attack applies more generally, covering quadratic regret among others. Assuming that the cost of sampling is c per observation where $c \rightarrow 0$, a relatively simple-minded study will indicate that the main contribution of the Bayes risk is due to values of μ

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