A MATHEMATICAL FORMULATION OF VARIATIONAL PROCESSES OF ADAPTIVE TYPE

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1. Summary

The questions we shall discuss in what follows belong to two fields which formerly were quite disjoint, the classical theory of probability and the classical calculus of variations. That there is now considerable overlap is due to the rise in scientific interest in the field of control processes. Although it is only within the last few years that the theory of feedback control has penetrated the academic curriculum and become a respectable member of the mathematical community, the conventional formulation is already far outmoded. In order to treat current and future problems of any significance, it is absolutely essential to introduce stochastic elements. These, however, enter in entirely novel ways, not in the fairly well understood fashion of conventional stochastic processes, but in connection with "learning processes" (compare [2]), or, as we shall henceforth say, *adaptive processes.*

In what follows we show how the functional equation technique of dynamic programming can be used to treat adaptive control processes, and how continuous processes can be defined in terms of the discrete versions.

2. Introduction

In order to prepare a suitable background for the introduction of the new features, let us review the elementary ideas of feedback control processes. We are, of course, here interested only in the mathematical presentation of these concepts, and shall ignore any of the difficulties of engineering or statistical application.

One version of the feedback control problem is that of maximizing a functional of the form

$$J(y) = \int_0^T F(x, y)\, dt \tag{2.1}$$

over all functions $y(t)$, where x and y are connected by means of a differential equation

$$\frac{dx}{dt} = G(x, y), \qquad x(0) = c, \tag{2.2}$$