

# STOCHASTIC LEARNING MODELS

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## 1. Introduction

Four kinds of studies of learning might reasonably be discussed under the title of this paper: (1) mathematical research on the theory of neuron networks, (2) the design of self-organizing mechanisms such as robots or computing machines [16], (3) the parts of information and communication theory that fall in the field of statistical behavioristics [29], (4) stochastic learning models for simple psychological experiments. This paper deals with the fourth topic.

There is a small but growing body of literature on statistical models constructed to assist experimental psychologists in the design, analysis, and explanation of some comparatively simple trial-by-trial learning experiments carried out under highly controlled conditions. In these experiments the response is either classified categorically or given as a time measure.

Because these models emphasize both the step-by-step process of learning and its statistical features, problems of time dependence, statistical estimation, and occasionally problems in theoretical probability arise. Thus far, sufficiently little work both of an experimental and theoretical nature has been done on the models and their extensions that there is still considerable unity in the publications. Furthermore the notions involved are quite elementary.

In this brief discussion two general categories of mathematical learning models have been omitted. Thurstone [35] develops learning curves initially from an urn scheme, but turns from this probabilistic model to differential equations. Similarly Gulliksen [20] and Gulliksen and Wolfe [21] and many others before and since work from differential equations, rather than from the kind of trial-by-trial models that are principally discussed below. On the other hand, Hull's extensive work (for one example see [26]) has been omitted, though it is sometimes related to the models presented here, because his postulational system would require a review of its own.

Finally, Savage's theory of personal probability [32] can, as he points out (p. 44), be regarded as a device for giving expression to the phenomenon of learning by experience. He also notes that logic itself "can be interpreted as a crude but sometimes handy empirical psychological theory" (p. 20). Such theories are omitted on the grounds that they are oriented more normatively than empirically and would not be likely to describe well the kind of behavior emitted in the experiments discussed here.

## 2. Beginning notions

For simplicity a situation with two response classes is discussed first, together with the form of the operator used to change the response probabilities. In section 3, a generalization to more than two response classes is introduced. Discussion there helps explain the choice of the form of the operators used.

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