SEQUENTIAL PROCEDURES FOR SELECTING THE BEST EXPONENTIAL POPULATION

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1. Introduction

A sequential procedure is considered for selecting that particular one of k exponential populations with the largest expected life (or the smallest failure rate). A. Birnbaum recently treated this problem [2], [3]. This paper extends his problem in two directions:

(i) The known guarantee period A_i, before which no items of the ith population fail, can be zero or of positive duration.

(ii) The number of populations k is assumed to be two or more.

In [3] it is assumed that $A_i = 0$ and $k = 2$. The case in which all populations have the same unknown guarantee period A is also treated. The effect of proceeding as if $A = 0$ when actually $A > 0$ is studied.

In life-testing experiments the problem is to utilize information provided by early failures to make decisions without waiting for all the units on test to fail. Frequently, it happens that even the expected lifetime of a single unit is longer than the experimenter is willing to wait before reaching a decision.

We shall assume in this paper that failure is a well defined and clearly recognizable phenomenon. We are interested in comparing the expected life (or the failure rate) of two or more populations. There are two classes of methods of reducing the time needed to reach a decision. One class consists of physical methods of accelerating the rate of failure (without introducing new causes of failure). The other class, with which this paper is concerned, consists of statistical methods.

One statistical method is to increase the initial number of units on test. Another is to replace each failure immediately by a new unit. A third is to use an appropriate *sequential* procedure to reach a decision. We shall consider some procedures which embody all three of the above features.

2. Definitions and assumptions

Suppose there are given k populations $\Pi_i (i = 1, 2, \cdots, k)$ such that the lifetimes of components taken from Π_i are distributed according to the delayed exponential density

$$f(x; A_i, \mu_i) = \frac{1}{\mu_i} e^{-(x-A_i)/\mu_i} \qquad \text{for } x \geqq A_i \tag{1}$$

(and $f = 0$ elsewhere), where the scale parameters $\mu_i > 0$ are assumed to be unknown, and also, since the location parameter A_i denotes a time delay, we shall assume that $A_i \geqq 0 (i = 1, 2, \cdots, k)$. Let the *ordered values* of the scale parameter μ be denoted by

$$\theta_1 \geqq \theta_2 \geqq \cdots \geqq \theta_k . \tag{2}$$