THE ROLE OF SUBJECTIVE PROBABILITY AND UTILITY IN DECISION-MAKING

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1. Introduction

Although many philosophers and statisticians believe that only an objectivistic theory of probability can have serious application in the sciences, there is a growing number of physicists and statisticians, if not philosophers, who advocate a subjective theory of probability. The increasing advocacy of subjective probability is surely due to the increasing awareness that the foundations of statistics are most properly constructed on the basis of a general theory of decision-making. In a given decision situation subjective elements seem to enter in three ways: (i) in the determination of a utility function (or its negative, a loss function) on the set of possible consequences, the actual consequence being determined by the true state of nature and the decision taken; (ii) in the determination of an *a priori* probability distribution on the states of nature; (iii) in the determination of other probability distributions in the decision situation.

These subjective factors may be illustrated by a simple example. A field general knows he is faced with opposing forces which consist of either (s_1) three infantry divisions and one armored division, or (s_2) two infantry divisions and two armored divisions. Thus the possible states of nature are s_1 and s_2. The possible consequences are a tactical victory (v), a stalemate (t), and a defeat (d). He subjectively estimates utilities as follows: $u(v) = 3, u(t) = 2, u(d) = -1$. On the basis of his intelligence he subjectively estimates the probability of s_1 as $\frac{1}{3}$, and of s_2 as $\frac{2}{3}$. Also in his view there are two major possible dispositions of his forces (f_1 and f_2). Using military experience and knowledge he now estimates the probability of victory, stalemate or defeat if he decides for disposition f_1 and s_1 is the true state of nature. Corresponding estimates are made for the pairs (f_1, s_2), (f_2, s_1) and (f_2, s_2). He then presumably decides on f_1 or f_2 depending on which yields the greater expected utility with respect to his estimated *a priori* distribution on s_1 and s_2.

In connection with this example, it may properly be asked why probabilities and utilities play such a prominent role in the analysis of the general's problem. The most appropriate initial answer, it seems to me, is that we expect the general's decision to be rational in some definite sense. The probabilities are measures of degree of belief, and the utilities measures of value. To be rational he should try to maximize expected value or utility with respect to his beliefs concerning the facts of the situation. The crucial problem is: what basis is there for introducing numerical probabilities and utilities? Clearly methods of measurement and a theory which will properly sustain the methods

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