DETERMINISTIC AND STOCHASTIC EPIDEMICS IN CLOSED POPULATIONS

DAVID G. KENDALL
MAGDALEN COLLEGE, OXFORD UNIVERSITY

1. Introduction

The problem of the growth of a stochastic epidemic in a closed population is a very challenging one; from one point of view it is almost trivial, for we have only to deal with a temporally homogeneous Markov process having a *finite* number of states, and yet there is great difficulty in finding out anything useful about the sample "epidemic curve" (the plot of the number of infectious individuals in circulation in the population as a function of the time[1]). We are much better informed about the ultimate behaviour of the system; the ordinate of the epidemic curve must eventually fall to zero and then the system will be "frozen" with say $X(\infty)$ "susceptibles" and $Z(\infty)$ "removed" persons, where $X(\infty) + Z(\infty)$ is equal to the (fixed) population size, and the form of the distribution of the random variable $Z(\infty)$ has been investigated by McKendrick [9] and by Bailey [1], [2]; the latter has even given an explicit formula for this distribution (due to F. G. Foster). So we know the ultimate fate of such a system, but little about how that state is attained. The difficulties blocking the way to an analytical solution are made clear by Professor Bartlett in his contribution to the present Symposium. In this paper I wish to sketch some approximate procedures which give a partial answer, which may be of value in more complicated situations of the same sort and which it would be interesting to replace by a mathematical argument. It is to be hoped that no one will apply them without reflexion, for there is nothing to be said in their favour beyond a certain intuitive plausibility and the questionable confirmation afforded by an armchair Monte Carlo experiment.

2. Deterministic epidemics

We must begin by making a thorough study of the corresponding *deterministic* system; this is the closed epidemic in the form in which it was studied by Kermack and McKendrick [8]. These writers partitioned the population of n individuals into three classes:

x susceptibles,
y infected persons in circulation in the population, and
z "removed" persons (recovered, dead or isolated),

so that

(1) $$x + y + z = n = \text{constant}.$$

[1] This definition of "epidemic curve" and the definition of the "notifications curve" in section 2 may differ from the reader's own usage.