ON THE STOCHASTIC THEORY OF EPIDEMICS

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1. Introduction

Early work in the mathematical theory of epidemics was mainly concerned with the development of deterministic models for the spread of disease through a population. An excellent review of the deterministic approach has been given by Serfling [15]. In this approach a functional equation (differential or integral equation, etc.) for $n(t)$, the number of infected individuals in the population at time t, is derived on the basis of certain assumptions concerning the mechanism by which the disease is to be transmitted among members of the population. This equation, together with some initial condition (the number of infected individuals at time zero), is then solved to obtain $n(t)$. In assuming a deterministic causal mechanism for the spread of an epidemic the number of infected individuals at some time $t > 0$ will always be the same if the initial conditions are identical. Because of the large number of random or chance factors which determine the manner in which an epidemic develops it became clear to workers in epidemic theory that probabilistic or stochastic models would have to be used to supplement or replace the existing deterministic ones.

The development of the theory of stochastic processes has given the mathematical epidemiologist the proper theoretical framework within which his mathematical models can be constructed. Of particular interest are stochastic processes of the branching or multiplicative type. These processes can be described as mathematical models for the development of systems whose components can reproduce, be transformed, and die; the development being governed by probability laws [9]. A discussion of some stochastic models in epidemic theory has been given by Taylor [16], and a detailed discussion of stochastic epidemic theory will be given in a monograph by the author [4]; hence, in this paper we will not give a review of previous work in this area. The purpose of the present paper is to consider the possible application of the Bellman-Harris theory of age-dependent branching processes [2] to epidemics, and to discuss some statistical problems associated with stochastic epidemics.

2. Age-dependent branching processes and epidemics

2.1. *Introduction.* In the Bellman-Harris theory the incubation period (defined as the length of time an individual is infected before infecting someone else) is a random variable, say τ, with general distribution $G(\tau)$, $0 < \tau < \infty$. At the end of this

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