FOUNDATIONS OF KINETIC THEORY

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1. Introduction

The basic equation of the kinetic theory of dilute monatomic gases is the famous nonlinear integro-differential equation of Boltzmann. In the simplest case when the molecules of the gas are hard spheres of diameter δ, which are allowed to exchange energy only through elastic collisions, the Boltzmann equation assumes the form

$$(1.1)\quad \frac{\partial}{\partial t} f(\vec{r}, \vec{v}, t) + \vec{v} \cdot \nabla_{\vec{r}} f + \vec{X}(\vec{r}) \cdot \nabla_{\vec{v}} f = \frac{\delta^2}{2} \int d\vec{w} \int d\vec{l}$$
$$\cdot \{ f(\vec{r}, \vec{v} + (\vec{w} - \vec{v}) \cdot \vec{l}\vec{l}, t) f(\vec{r}, \vec{w} - (\vec{w} - \vec{v}) \cdot \vec{l}\vec{l}, t) - f(\vec{r}, \vec{v}, t) f(\vec{r}, \vec{w}, t) \}$$
$$\cdot |(\vec{w} - \vec{v}) \cdot \vec{l}|;$$

here $f(\vec{r}, \vec{v}, t) d\vec{r}\, d\vec{v}$ is the average number of molecues in $d\vec{r}\, d\vec{v}$ at $\vec{r}, \vec{v}$, $\nabla_{\vec{r}} f$ the gradient of f with respect to $\vec{r}$, $\nabla_{\vec{v}} f$ the gradient of f with respect to $\vec{v}$, $\vec{l}$ a unit vector and $d\vec{l}$ the surface element of the unit sphere. $\vec{X}(\vec{r})$ is an outside force (for example, gravity) acting on a particle at $\vec{r}$. If the gas is enclosed in a container of volume V and if there are no exterior forces ($X(\vec{r}) \equiv 0$) we can set

$$(1.2)\qquad f(\vec{r}, \vec{v}, t) = \frac{n}{V} f(\vec{v}, t),$$

where n is the total number of molecules, and note that it will be a solution of (1.1) if $f(\vec{v}, t)$ is a solution of the reduced Boltzmann equation

$$(1.3)\qquad \frac{\partial}{\partial t} f(\vec{v}, t) = \frac{n\delta^2}{2V} \int d\vec{w} \int d\vec{l}\, \{ f(\vec{v} + (\vec{w} - \vec{v}) \cdot \vec{l}\vec{l}, t) f(\vec{w} - (\vec{w} - \vec{v}) \cdot \vec{l}\vec{l}, t)$$
$$- f(\vec{v}, t) f(\vec{w}, t) \} |(\vec{w} - \vec{v}) \cdot \vec{l}|.$$

Equation (1.3) governs the temporal evolution of the velocity distribution while the spatial distribution remains uniform.

If the molecules are not hard spheres but are considered as centers of force,

$$(1.4)\qquad \frac{\delta^2}{2} |(\vec{w} - \vec{v}) \cdot \vec{l}|$$

has to be replaced by an expression depending on the nature of the force.

The most famous example is that of a Maxwell gas in which the molecules are

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