STATISTICAL MECHANICS AND PROBABILITY THEORY

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1. Introduction

As pointed out by Khinchin [1], statistical mechanics presents two fundamental classes of problems for mathematics:

(a) the problems which are closely connected with the ergodic theory, and

(b) the problems which stem from the fact that the systems considered have many degrees of freedom.

The latter problems are concerned with the creation of an analytic method for the construction of asymptotic formulas.

The present work deals only with the problems of the second class. In order to describe the macroscopic properties of a given mechanical system composed of a very large number of particles with negligible interaction, one often defines the so-called most probable macroscopic state, namely, the distribution of particles (in their different possible states) having the largest probability of realization.

The mathematical expression for the probability of a macroscopic state uses formulas of combinatorial analysis which contain the factorial function, and, in the determination of the most probable state, Stirling's approximation [2] of $N!$. This method is relatively simple but it is not rigorous. Moreover, it is necessary to show in a rigorous way why the most probable state is indeed characteristic of macroscopic properties. The most probable value is not sufficient to determine all the properties of a random variable. However, under rather general conditions, the set of all the moments characterizes the probability distribution. Fowler makes use of the method of steepest descent to determine the values of all moments [3]. However, when he devised his analytical method, the theory of probability was not so developed as it is today. Hence, Fowler did not use certain methods of the theory of probability which, as we shall see, are particularly convenient and efficient. The main problems of statistical mechanics can be reduced to certain fundamental questions of probability theory. This procedure avoids the use of the certainly ingenious but often artificial tools used by Fowler. A number of papers have been published concerning this reduction of statistical mechanics to the theory of probability. A basic point is the following: *the problems of statistical mechanics can be reduced to classical problems of conditional probability*. For instance, it may be necessary to know the statistical properties of a system under the condition that its energy is supposed given.

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