L-RANDOM ELEMENTS AND L^*-RANDOM ELEMENTS IN BANACH SPACES

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1. Introduction

It is well known that many applications of the theory of probability require the consideration of general random elements. We are mainly interested in generalizing laws of large numbers and central limit theorems, so we must consider random elements in topological vector spaces. As a first approach we study random elements in Banach spaces and in the first part of this paper give the definitions and results we have obtained (see [1] and [9]). The case of Banach spaces may seem very limited; however, we show in the second part that it allows many applications. Finally, in the last part, we indicate a new point of view and the results we have established in this way.

2. Definition of L-random elements in Banach spaces and their mathematical expectation

Let $(U, \mathcal{Q}, m)$ be a fundamental probability space of elements u, let $\mathfrak{X}$ be a Banach space of elements x, and $x(u)$ a function on U to $\mathfrak{X}$. We call $X = x(u)$ a *random element* in $\mathfrak{X}$, that is, the "value" of the random element X is $x(u) \in \mathfrak{X}$ if the outcome of the experiment is $u \in U$.

Let $\mathfrak{X}^*$ be the dual space of $\mathfrak{X}$, that is, the space of all continuous linear functionals x^* on $\mathfrak{X}$. We shall write $\langle x^*, x\rangle$ for the number obtained by applying the linear functional $x^* \in \mathfrak{X}^*$ to $x \in \mathfrak{X}$. The *mathematical expectation* of X is the element $E(X)$ of $\mathfrak{X}$, if one exists, such that, for all $x^* \in \mathfrak{X}^*$,

$$\langle x^*, E(X) \rangle = E(\langle x^*, X\rangle). \tag{2.1}$$

If $E(X)$ exists, it is unique [6], and $E(X)$ is the Pettis integral [11], $\int_U x(u)dm$, of $x(u)$ with respect to the measure m.

Such a definition of mathematical expectation implies that, for all $x^* \in \mathfrak{X}^*$, $\langle x^*, x(u)\rangle$ is a measurable function of u. We call *L-random elements* those random elements for which this property is fulfilled, and in this section we shall consider only L-random elements.

It is easy to prove (see [9]) the following:

(a) If a is a given number and if $E(X)$ exists, then $E(aX)$ exists and $E(aX) = aE(X)$.

(b) If X is almost surely (a.s.) equal to a given element x, then $E(X)$ exists and $E(X) = x$.

(c) If X is almost surely (a.s.) equal to a given element x, and if A is a random variable such that $E(A)$ exists, then $E(AX)$ exists and $E(AX) = xE(A)$.

This paper was prepared with partial support of the Office of Naval Research.