FOUNDATIONS OF THE THEORY OF CONTINUOUS PARAMETER MARKOV CHAINS

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1. Foreword

This paper is exclusively concerned with continuous parameter Markov processes with a denumerably infinite number of states and stationary transition matrix function. The foundations of the proper theory of such processes, as distinguished from that of the discrete parameter version, or of Markov processes which are either more special (for example, a finite number of states) or more general (for example, general state space; nonstationary transition matrix function), were laid by Doob [1], [2], and Lévy [4], [5], [6].[1] Roughly speaking it was Lévy in 1952 who drew, in his inimitable way, the comprehensive picture while Doob, ten years earlier, had supplied the essential ingredients. The present effort aims at a synthesis of the most fundamental parts of the theory, made possible by the contributions of these two authors. While the results given here generally extend and clarify those in the cited literature, immense credit must go to Professors Lévy and Doob for the inspiration of their pioneer work. To them I am also indebted for much valuable discussion through correspondence and conversation. An attempt is made in the presentation to be quite formal and rigorous, in the spirit of Doob's already classic treatise [3]. Further developments of the theory will be published elsewhere.

2. Introduction

We consider a probability space Ω with the generic point ω, a Borel field $\mathcal{B}$ of ω-set including Ω itself, and a (complete) probability measure P defined on $\mathcal{B}$. For general definitions and notations we refer to [3], unless otherwise specified. The notation $x(t, \omega)$, for example, will be used both for the function $x(t, \cdot)$ and its value at ω.

A Markov chain $\{x(t, \omega), 0 \leqq t < \infty\}$ is given as follows. The state space is the set of nonnegative integers. The initial distribution is given by

$$P\{x(0, \omega) = i\} = p_i, \qquad i = 0, 1, 2, \cdots \tag{2.1}$$

where $p_i \geqq 0, \sum p_i = 1$. The stationary transition probability functions are

$$p_{ij}(t) = P\{x(s+t, \omega) = j \mid x(s, \omega) = i\}, \qquad s \geqq 0, t > 0, \tag{2.2}$$

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[1] It is a pleasure to note that Professor Lévy in his paper [4] attributed its origin to a conversation held in the course of the Second Berkeley Symposium.