SOME REGRESSION PROBLEMS IN TIME SERIES ANALYSIS

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1. Introduction

Estimates of the regression coefficients which are unbiased and linear in the observations are discussed in this paper. The residual is assumed to be a stationary process. Two specific estimates are discussed, the least-squares estimate and the Markov estimate. I call the estimate which is computed under the assumption that the residual is an orthogonal process the least-squares estimate. The Markov estimate is the linear unbiased estimate with minimal covariance matrix. The basic assumptions made in the paper are discussed in section 2 and are held to throughout the paper. In section 3 some remarks about the approximation of a continuous positive definite matrix-valued function by finite trigonometric forms are made. These remarks are used in section 4 to obtain the main results about the asymptotic behavior of the covariance matrices of the least-squares and Markov estimates. The next section discusses the many interesting cases in which the least-squares estimate is asymptotically as good as the Markov estimate. The first really systematic discussion of some of these problems was given by U. Grenander [1]. Further work was carried out by U. Grenander and M. Rosenblatt in [2], [3], and [4]. The author considers some of these problems in the case of a vector-valued time series in [5]. Some of the results of this paper are a generalization of some of those obtained in [5].

A few cases in which the least-squares estimate is not asymptotically efficient in the class of linear unbiased estimates are discussed in sections 5 and 7. Some small sample computations for a linear regression with a residual which is a first order autoregressive scheme are carried out in section 6 to test the asymptotic theory.

2. Assumptions and notation

I assume that the observed process y_t is a vector-valued process (a k-vector)

$$y_t = x_t + m_t, \qquad t = \cdots, -1, 0, 1, \cdots, \tag{2.1}$$

where $m_t = Ey_t$ is the mean value sequence and x_t, $Ex_t \equiv 0$, is the sequence of residuals. The residual x_t is assumed to be weakly stationary, that is, the covariances

$$r_{t\ \tau} = r_{t-\tau} = Ex_t x'_\tau = E(y_t - m_t)(y_\tau - m_\tau)' \; {}^{2} \tag{2.2}$$

depend only on the difference $t - \tau$. For mathematical convenience, in sections 3 and 4, I assume that the components of the vector observations are complex valued. The real-

Based in part on research supported by the Office of Naval Research at the Statistical Research Center, University of Chicago.

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[2] x_t is column vector. Given a matrix A, A' denotes the conjugated transpose of A.