ON THE ASYMPTOTIC THEORY OF ESTIMATION AND TESTING HYPOTHESES

L. LE CAM
UNIVERSITY OF CALIFORNIA

1. Introduction

The purpose of the present paper is to develop a systematic method of reduction of problems of estimation and testing hypotheses to similar problems on normal distributions. The method proposed is only valid asymptotically and but little is known about its performance for samples of finite size.

To describe more precisely the questions investigated it would be necessary to proceed to an historical review of large sample theory. For brevity and simplicity we shall restrict ourselves to the key papers of J. Neyman and A. Wald. The first of these authors introduced in [1] what are called best asymptotically normal regular estimates (B.A.N. for short). The situation considered by Neyman is one in which the distributions are of a multinomial nature, but the same techniques apply to families of the Koopman-Darmois type (see [2]). Part of the motivation for the introduction of B.A.N. estimates is that the maximum likelihood estimates are, even in such a "simple" case, very often difficult to obtain. Furthermore, the B.A.N. estimates behave asymptotically very much like the maximum likelihood estimates.

In sharp contrast with the preceding, Wald [3] considers classes of densities restricted only by regularity conditions. In such a case, sufficient statistics of fixed dimensionality do not usually exist, so that the methods used by Wald are by necessity different from those of Neyman. Wald confines his attention to maximum likelihood estimates and tests based on these estimates. A fundamental result of Wald is that, under certain conditions, the maximum likelihood estimates are "asymptotically sufficient." Further, by means of suitable *set* transformations it is possible to associate to each test problem on the original distributions a closely related, though not equivalent, problem on normal distributions. The asymptotic sufficiency of the maximum likelihood estimates would make Neyman's techniques available to the statistician if only he could obtain the values of the estimates. Since Wald's reasoning relies heavily on the fact that maximum likelihood estimates are roots of the corresponding equations, it is not at all clear that the same results would remain valid for approximate maximum likelihood estimates. It is even less clear that the results would hold when the maximum likelihood estimates are not solutions of the relevant equations, a circumstance which occurs often on boundaries of the parameter space.

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