THE ROLE OF ASSUMPTIONS IN STATISTICAL DECISIONS

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1. Introduction

In order to obtain a good decision rule for some statistical problem we start by making assumptions concerning the class of distributions, the loss function, and other data of the problem. Usually these assumptions only approximate the actual conditions, either because the latter are unknown, or in order to simplify the mathematical treatment of the problem. Hence the assumptions under which a decision rule is derived are ordinarily not satisfied in a practical situation to which the rule is applied. It is therefore of interest to investigate how the performance of a decision rule is affected when the assumptions under which it was derived are replaced by another set of assumptions.

We shall confine ourselves to the consideration of assumptions concerning the class of distributions. Investigations of particular problems of this type are numerous in the literature. There are many studies of the performance of "standard" tests under "non-standard" conditions, for example [3], where further references are given. Most of them considered only the effect of deviations from the assumptions on the significance level of the test. The relatively few studies of the effect on the power function include several papers by David and Johnson, the latest of which is [6]. For some problems tests have been proposed whose significance level is little affected by certain deviations from standard assumptions, for instance R. A. Fisher's randomization tests (see section 3; see also Box and Andersen [4]). Some other relevant work will be mentioned later.

In sections 2, 3, and 4 we shall be concerned with problems of the following type. Let P denote the joint distribution of the random variables under observation. Suppose that we contemplate making the assumption that P belongs to a class $\mathcal{P}_1$, but we admit the possibility that actually P is contained in another class, $\mathcal{P}_2$. The performance of a decision rule (decision function) d is assumed to be expressed by the given risk function $r(P, d)$, defined for all $P \in \mathcal{P}_1 + \mathcal{P}_2$ and all d in D, the class of decision rules available to the statistician. Let d_i be a decision rule which is optimal in some specified sense (for example, minimax) under the assumption $P \in \mathcal{P}_i$, $i = 1, 2$. Suppose first that the optimal rule d_i is unique except for equivalence in $\mathcal{P}_1 + \mathcal{P}_2$, for $i = 1, 2$, that is, if d_i' is also optimal for $P \in \mathcal{P}_i$ then $r(P, d_i') = r(P, d_i)$ for all $P \in \mathcal{P}_1 + \mathcal{P}_2$. Then we may assess the consequences of the assumption $P \in \mathcal{P}_1$ when actually $P \in \mathcal{P}_2$ by comparing the values $r(P, d_1)$ and $r(P, d_2)$ for $P \in \mathcal{P}_2$. If the optimal rules are not unique, we may pick out from the class of rules which are optimal for $P \in \mathcal{P}_1$ a subclass of rules which come closest to optimality under the assumption $P \in \mathcal{P}_2$, and compare their performance with that of the rules which are optimal under the latter assumption. In

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