SOME PROBLEMS IN ESTIMATING THE SPECTRUM OF A TIME SERIES

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1. Introduction

We consider a probability model that has proved to be useful in various applied fields. Statistical problems that arise in the analysis of data obtained in these fields are discussed. The basic model is that of a stochastic process $\{y_t\}$

$$y_t = x_t + m_t \tag{1.1}$$

where $m_t = Ey_t$ is the mean value of y_t and x_t, $Ex_t \equiv 0$, is the residual. The residual x_t is assumed to be stationary with respect to the parameter t, that is,

$$x_{t_1}, \cdots, x_{t_n} \tag{1.2}$$

have the same probability distribution as

$$x_{t_1+h}, \cdots, x_{t_n+h} \tag{1.3}$$

for all possible values of $t_1, \cdots, t_n, h$. In other words, the probability distribution of x_t is invariant under t displacement. This implies that the set of possible values of t, which we shall call T, is a group or semigroup under addition. Typical examples of the parameter set T are the set of all points in Euclidean k-space or the set of lattice points in Euclidean k-space. These are in fact the examples of greatest interest and they will be discussed in some detail.

The process $\{y_t\}$ may be vector valued. An example of interest in which the vector-valued case is appropriate will be described.

A usual situation is that in which t is thought of as time. The parameter t will then be a point of the form kh if the observations are taken at discrete time points with h seconds between each observation. If the observation is continuous, t will be any real number.

The case in which $m_t \equiv 0$ is of considerable importance. Such a model is appropriate where the phenomenon studied consists of random fluctuations which are of a stable character.

Some of the fields in which such a model has been used will be discussed in section 2. These fields are in the physical sciences. They are discussed to give some motivation to

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