

THE ESTIMATION OF THE LOCATION OF A DISCONTINUITY IN DENSITY

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1. Introduction

It is well known that the maximum likelihood estimates of the parameters of a rectangular distribution are extremely efficient. In fact, these estimates differ from the parameter by an amount which is of the order of magnitude of $1/n$ where n is the sample size. In this paper a similar result is obtained for the maximum likelihood estimate of a where a is one of a set of parameters determining the family of distributions and a is the location of a point of discontinuity of the density of the random variable observed. The estimate \hat{a} is approximately one of the observations in the neighborhood of a .

The limiting distribution of $n(\hat{a} - a)$ is related to a random walk problem. One part of this random walk problem involves the distribution of $\sum_{i=1}^R z_i$ where R is the value of r

which minimizes $\sum_{i=1}^r (z_i - \omega)$, the z_i are independent and have the exponential distribution with mean 1 and $0 < \omega < 1$. The limiting distribution depends only on β and γ where these are the one-sided limits of the density at a .

This type of problem arises naturally whenever one considers a population which has two subpopulations and one of these is truncated. For example, a teacher may have students who can afford to pay tuition or those who cannot pay but have obtained a scholarship on the basis of a mark in a competitive examination. If he does not know the passing mark or which students have scholarships, the problem of estimating the passing mark is of the above type.

Consider the family of distributions given by the density

$$(1) \quad \begin{aligned} f(x, a, \beta, \gamma) &= \beta, & 0 \leq x \leq a, \\ f(x, a, \beta, \gamma) &= \gamma, & a < x \leq 1 \end{aligned}$$

where

$$(2) \quad \beta a + \gamma(1 - a) = 1,$$

and $0 < a < 1$.

This paper was written with the partial support of the Office of Naval Research, and the Office of Ordnance Research, U.S. Army, under Contract DA-04-200-ORD-355.