# ON A USE OF THE MANN-WHITNEY STATISTIC

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## 1. Introduction

Let $X$ and $Y$ be independent random variables with continuous cumulative probability functions $F$ and $G$, respectively, and let

$$X_1, X_2, \cdots, X_m; \qquad Y_1, Y_2, \cdots, Y_n \tag{1.1}$$

be samples of $X$ and of $Y$. Mann and Whitney [1] considered the statistic

$$U = \text{number of pairs } (X_j, Y_k) \text{ such that } Y_k < X_j . \tag{1.2}$$

For $m = n$ an equivalent statistic had been proposed and studied earlier by Wilcoxon [2]. The main aim of these studies was to develop a test of the hypothesis that $X$ and $Y$ have the same probability distribution: $F = G$. More about this test will be reported in section 2.

Independently, Haldane and Smith [3] investigated the following problem. In some hereditary conditions, the probability that a member of a sibship has the condition depends partly on his birth rank. Having records of sibships in the order of birth, stating for each individual whether it has or does not have the condition, how does one test for independence of the condition from birth rank? To answer this question, Haldane and Smith constructed a test statistic which is equivalent with the $U$ statistic (1.2).

Without an attempt at completeness, we shall give in section 2 a brief survey of the known properties of the $U$ statistic which are of importance for its use in testing hypotheses. The main purpose of the present paper, however, is to discuss another use of this statistic which, while not new, seems to have attracted less attention. Let

$$p = Pr\{Y < X\}. \tag{1.3}$$

If the samples (1.1) are available, then the statistic

$$\hat{p} = \frac{U}{mn} \tag{1.4}$$

can be used to estimate the parameter $p$. It is this particular use of $U$ which will be explored in some detail in sections 3 and 4.

## 2. Properties of $U$ useful in testing hypotheses

Under the hypothesis $(H)$: $F = G$, Mann and Whitney [1] have tabulated the exact probability distribution of $U$ for $m \leqq n \leqq 8$, and proved that

$$\frac{U - \frac{1}{2}mn}{\sqrt{mn(m+n+1)/12}} \tag{2.1}$$

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