CORRELATION OF POSITION FOR THE IDEAL QUANTUM GAS

G. PLACZEK
INSTITUTE FOR ADVANCED STUDY

1. Introduction*

It is well known that in quantum theory the particles of a gas cannot be considered as independent even in the limiting case of zero interaction (ideal gas). As a consequence of the symmetry properties of the wave function, the space coordinates of the particles are correlated in the sense of an apparent repulsion in the case of Fermi-Dirac statistics and of an apparent attraction in the case of Bose-Einstein statistics. The explicit quantitative form of the correlation has been given for temperatures high compared to the degeneracy temperature [1] and for the fully degenerate Fermi gas [2]. In the following we shall discuss the general case of the ideal gas without interaction and the connection between correlation of position, density fluctuation and scattering properties of the gas.

The procedure followed is a slight generalization of a method used by Heisenberg [3].

2. General relations

We consider the density in six dimensional coordinate space (pair density) which we denote by $\rho_2(\bar{r}, \bar{r}')$. For a gas, this quantity can only be a function of the absolute value of the distance between the particles and may hence be written

$$\rho_2(\bar{r}, \bar{r}') = \rho^2 W(|\bar{r} - \bar{r}'|) . \tag{1}$$

Here ρ is the ordinary density (number of particles per unit volume) and $\rho W(R)$ the density at distance R from a given particle. If the particles are independent, $W(R) = 1$; in the general case, $W(R)$ tends to 1 for large R. The function $W(R)$ may be related to the density fluctuations as follows [4], [5]. Denoting by z_i and z_k the numbers of particles contained in infinitely small volume elements dv_i, dv_k located at distance R_{ik} from each other, we have, since the probability that the numbers z_i and z_k take values other than zero or one is negligible:

$$\begin{cases} (z_i^2)_{Av} = (z_i)_{Av} = \rho\, d v_i \,, \\ (z_i z_k)_{Av} = \rho^2 W(R_{ik})\, d v_i d v_k \,. \end{cases} \tag{2}$$

* *Added in proof:* Practically all the expressions for the correlation function $W(R)$ given in the present paper have already been obtained in a paper by F. London [8] which had escaped my attention. London's results have recently been rederived by G. Leibfried [9]. The only result concerning $W(R)$ which goes essentially beyond the contents of these papers is its asymptotic form (36).