STATISTICAL MECHANICS OF A CONTINUOUS MEDIUM (VIBRATING STRING WITH FIXED ENDS)

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1. For the past twenty years there has been much discussion in fluid mechanics about the statistical theory of turbulence. It was J. Boussinesq (1872) and O. Reynolds (1895), in their pioneering work, who expressed the idea that the turbulent velocity fluctuations of a fluid were much too complicated (changing rapidly from one time and one point to another) to be known in all their details; we must be satisfied to study only some convenient mean values. The systematic use of statistical methods has led, since 1930, to very important results, in the fundamental works of Sir Geoffrey Taylor, Th. von Karman and A. N. Kolmogoroff, for instance.

But, if we look carefully at all the results so far obtained, we see at first glance that they are not at all in the same close relation with the theoretical equations of fluid mechanics as the classical statistical mechanics bears to the Hamilton-Jacobi equations for a dynamical system having a finite number of degrees of freedom. For such a system the main features of statistical mechanics are the following:

(a) definition of the phase space Ω; every state of the system is characterized by a point $\omega \in \Omega$, whose $2k$ coordinates are the Lagrangian parameters $q_1, \ldots, q_k$ and the conjugate moments $p_1, \ldots, p_k$.

(b) proof of a uniqueness theorem: starting, at the initial time $t = 0$, from a given initial state ω, all the subsequent or prior states $T_t \omega$, where t varies from $-\infty$ to $+\infty$, are perfectly well determined and describe, in Ω, a curve, the trajectory Γ_ω.

(c) definition of a measure in Ω, invariant under the transformation $T_t \omega$. (Liouville's theorem.)

(d) proof of the ergodic theorem (existence of time averages computed along a trajectory Γ_ω); the time averages and statistical averages can be validly considered as equal only in the special case in which the transformation $T_t \omega$ of Ω into itself is metrically transitive.

As we have pointed out in [4], at the time being, all the work done in what is called statistical theory of turbulence deals only with some kind of time averages; so far as we know, a convenient phase space Ω has never been introduced, in order to represent all the possible states ω of a fluid; this phase space must be, as we have shown in [5], a function space, quite easy to define. But, so little is known about the solutions of the nonlinear equations of fluid mechanics (for a perfect as well as for a viscous fluid), that immediately after the definition of Ω, we are stopped by