RANDOM FUNCTIONS FROM A POISSON PROCESS

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1. Definition of random functions from a Poisson process

Let $R(t, \tau)$ be an ordinary (nonrandom) function of two variables t and τ, which, for example, might represent time. Let us consider instants distributed at random on the time axis according to the classical Poisson process of density m ($m > 0$). Let $N_1(t)$ be the number of these instants belonging to the interval $(0, t)$ if $t > 0$, to the interval $(t, 0)$ if $t < 0$. We shall write $N(t) = N_1(t)$ if $t > 0$; $N(t) = -N_1(t)$ if $t < 0$, so that $N(t') - N(t)$ represents the number of instants belonging to (t, t') whatever may be t and t' $(t < t')$.

The following well known results should be recalled:

(a) $N(t)$ is a random function with independent increments, taking integer values, and is almost certainly nondecreasing.

(b) The probability that the number of instants belonging to any finite interval of time be infinite is zero.

(c) The distribution in the time of these instants is stationary.

(d) m is the expectation of the number of instants belonging to any interval of amplitude 1.

(e) The probability that the number of instants belonging to any interval (t, t'), $t < t'$, be equal to n is equal to

$$e^{-m\delta} \frac{(m\delta)^n}{n!},$$

where $\delta = t' - t$.

Calling $\tau_1, \tau_2, \ldots, \tau_j, \ldots$ the instants of an interval (t_0, t), $(t > \tau_0)$, different applications, for example, the phenomenon of noise in electronics, lead to a consideration of random functions of t defined in the following way:

$$X(t) = \sum_j R(t, \tau_j), \tag{1}$$

which may be written as

$$X(t) = \int_{t_0}^{t} R(t, \tau)\, dN(\tau). \tag{2}$$

More generally, we shall consider

$$X(t) = \int_{-\infty}^{+\infty} R(t, \tau)\, dN(\tau), \tag{3}$$

and we shall denote random functions of this kind, that is random functions from a Poisson process, by P.r.f. The study of these functions has been developed by