A CONTRIBUTION TO THE THEORY OF STOCHASTIC PROCESSES

HARALD CRAMÉR
UNIVERSITY OF STOCKHOLM

1. Introduction

Let ω denote a point or element of an arbitrary space Ω, where a probability measure $\Pi(\Sigma)$ is defined for every set Σ belonging to a certain additive class of sets in Ω, the Π-measurable sets. The probability distribution in Ω defined by $\Pi(\Sigma)$ will be referred to as the *probability field* (Π, Ω). The points ω will be denoted as the *elementary events* of the field, while any set Σ corresponds to an *event*, the probability of which is equal to $\Pi(\Sigma)$.

A complex valued Π-measurable function

$$x = g(\omega)$$

constitutes a *random variable*, defined on the field (Π, Ω). The *mean value* of x is defined by the relation

$$Ex = \int_{\Omega} g(\omega)\, d\Pi. \tag{1}$$

Throughout the paper, we shall always assume that, for every random variable considered, we have

$$Ex = \int_{\Omega} g(\omega)\, d\Pi = 0\,, \qquad E|x|^2 = \int_{\Omega} |g(\omega)|^2 d\Pi < \infty\,.$$

The first condition introduces some formal simplification, but does not imply any restriction of the generality of our considerations, while the second condition is essential. Two variables x and y are considered as identical, if $E|x - y|^2 = 0$.

Consider a complex valued function $x(t, \omega)$ such that, for every fixed t belonging to some specified set T, the function $x(t, \omega)$ is a Π-measurable function of ω, and thus defines a random variable $x(t)$ on the field (Π, Ω). When t ranges over T, we thus obtain a family of random variables, depending on the parameter t. On the other hand, to any fixed elementary event ω there corresponds a function

$$x(t) = x(t, \omega)\,,$$

defined for all t belonging to T, and to any event Σ there corresponds a set of functions $x(t)$ having the probability $\Pi(\Sigma)$. The function $x(t)$ will be denoted as a *random function*, defined on the field (Π, Ω).

Throughout this paper, the set T will be assumed to be the real axis, $-\infty < t < +\infty$. However, most of our considerations may easily be extended to more general spaces.