SOME MATHEMATICAL MODELS FOR BRANCHING PROCESSES

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1. Introduction

The stochastic processes variously called branching, birth, or multiplicative processes have been considered by writers in many different fields during the past eighty odd years. We shall not try to characterize such processes mathematically, although certain related mathematical properties will appear in all the processes we study. Physically speaking we may say that they represent the evolution of aggregates or systems whose components can reproduce, be transformed, and die, the transitions being governed by probability laws. The examples which have been most frequently considered in applications are the propagation of human and animal species and genes, nuclear chain reactions, and electronic cascade phenomena. The first, and probably best known mathematical model, which we shall consider in section 2, arose in connection with the problem of "the extinction of family surnames," and was treated by Galton and Watson [1] as far back as 1874.

As we should expect, the mathematical models which are simple enough to make possible a thorough analytic treatment of the subject are often radical oversimplifications of reality. Nevertheless, certain practical applications of the theory have been possible.

For a good historical account of the subject, including many references to applications, as well as interesting original work, we refer the reader to papers by M. S. Bartlett [2] and David G. Kendall [3]. Their bibliographies, together with that at the end of this paper, give (not completely exhaustive) references to much of what has been written in the field. It is unfortunate that some work done during the war, and classified, is still not available.

The present paper considers a number of stochastic processes which have been used as models for branching phenomena. We shall be particularly concerned with limiting theorems and limiting distributions giving the behavior of the systems studied after long periods of time. One pattern recurs often enough to make the following statement plausible, although a general mathematical formulation has not been given. It is strongly suggested by results of Everett and Ulam [4] and various results of Harris [5] and Bellman and Harris [6].

Consider a family of objects. Each object is described at a given instant of time by a vector quantity x, where x may describe the age, energy, position in space, or a combination of these or other traits. The quantity x for a given object may vary with time in a deterministic or a random fashion. In addition, there is a law for the probability that an object of "type" x, existing at time t, will produce (or be trans-