RANDOM ERGODIC THEOREMS AND MARKOFF PROCESSES WITH A STABLE DISTRIBUTION

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1. Introduction

The purpose of this paper is to discuss the relations between random ergodic theorems and Markoff processes with a stable distribution. Random ergodic theorem concerning a finite number of measure preserving transformations was obtained by S. M. Ulam and J. von Neumann. This result was announced in abstract form [6] but the proof has never been published. In the present paper we shall first give a proof of random ergodic theorem concerning a family of (infinitely many) measure preserving transformations with a probability distribution on it. We shall then discuss the condition of ergodicity for a family of measure preserving transformations and its consequence in random ergodic theorems. It turns out that the theory of Markoff processes with a stable distribution which was previously discussed by J. L. Doob [2], [3], K. Yosida [8], and the author [4] has a very close connection with our problem. It will be shown that to any family Φ of measure preserving transformations with a probability distribution there corresponds a Markoff process $P(s, B)$ with a stable distribution in such a way that the ergodic theorems concerning the Markoff process $P(s, B)$ which were obtained in [8] and [4] are nothing but the "integrated form" of random ergodic theorems concerning the family Φ of measure preserving transformations. Further, the conditions of ergodicity for P correspond exactly to those for Φ. It is, indeed, by making use of this fact that we prove the equivalence of various conditions of ergodicity for the family Φ of measure preserving transformations.

In case Φ consists of a finite number of measure preserving transformations, our ergodic theorem is reduced to that of S. M. Ulam and J. von Neumann [6]. If, in particular, the space on which the measure preserving transformations act is finite (and hence the measure preserving transformations are reduced to a permutation) our theory is reduced to that of H. Anzai [1] on the relationship between the random ergodic theorem concerning a finite number of permutations and the theory of Markoff process with a finite number of possible states.

We do not discuss it in our present paper, but it is an interesting problem to investigate the conditions of (weak or strong) mixing for a family Φ of measure preserving transformations and for the corresponding Markoff process P with a stable distribution.