ON MEDIAN TESTS FOR LINEAR HYPOTHESES

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1. Introduction and summary

This paper discusses the rationale of certain median tests developed by us at Iowa State College on a project supported by the Office of Naval Research. The basic idea on which the tests rest was first presented in connection with a problem in [1], and later we extended that idea to consider a variety of analysis of variance situations and linear regression problems in general. Many of the tests were presented in [2] but without much justification. And in fact the tests are not based on any solid foundation at all but merely on what appeared to us to be reasonable compromises with the various conflicting practical interests involved.

Throughout the paper we shall be mainly concerned with analysis of variance hypotheses. The methods could be discussed just as well in terms of the more general linear regression problem, but the general ideas and difficulties are more easily described in the simpler case. In fact a simple two by two table is sufficient to illustrate most of the fundamental problems.

After presenting a test devised by Friedman for row and column effects in a two way classification, some median tests for the same problem will be described. Then we shall examine a few other situations, in particular the question of testing for interactions, and it is here that some real troubles arise. The asymptotic character of the tests is described in the final section of the paper.

2. Friedman's test

In [3] Friedman presented the following test for column effects in a two way classification with one observation per cell. Letting r and s denote the number of rows and columns, the observations in each row are replaced by their ranks in the row; the rows then consist of permutations of the numbers 1, 2, . . . , s. Letting T_j be the column totals of these ranks, it is clear that the distribution of the T_j is independent of the form of the distribution of the observations provided the observations are continuously and identically distributed in rows. Thus under the null hypothesis of no column effects the quantity

$$\chi^2 = \frac{12}{rs(s+1)} \sum_{j=1}^{s} \left(T_j - \frac{r(s+1)}{2} \right)^2$$

is distribution free, and Friedman has shown that it has approximately the chi-square distribution with $s - 1$ degrees of freedom when r is large. Of course the hypothesis of no row effects may be tested by reversing the roles of rows and columns in the test criterion.