CHARACTERIZATION OF THE MINIMAL COMPLETE CLASS OF DECISION FUNCTIONS WHEN THE NUMBER OF DISTRIBUTIONS AND DECISIONS IS FINITE

A. WALD AND J. WOLFOWITZ
COLUMBIA UNIVERSITY

1. Introduction

The principal object of the present paper is to prove theorem 2 below. This theorem characterizes the minimal complete class in the problem under consideration, and improves on the result of theorem 1. Theorem 1 has been proved by one of us in much greater generality [1]. The proof given below is new and very expeditious. Another reason for giving the proof of theorem 1 here is that it is the first step in our proof of theorem 2. A different proof of theorem 1, based, like ours, on certain properties of convex bodies in finite Euclidean spaces, was communicated earlier to the authors by Dr. A. Dvoretzky. Theorem 3 gives another characterization of the minimal complete class.

Let x be the generic point of a Euclidean[1] space Z, and $f_1(x), \ldots, f_m(x)$ be any m (> 1) distinct cumulative probability distributions on Z. The statistician is presented with an observation on the chance variable X which is distributed in Z according to an unknown one of $f_1, \ldots, f_m$. On the basis of this observation he has to make one of l decisions, say $d_1, \ldots, d_l$. The loss incurred when x is the observed point, f_i is the actual (unknown) distribution, and the decision d_j is made, is $W_{ij}(x)$, where $W_{ij}(x)$ is a measurable function of x such that

$$\int_Z |W_{ij}(x)|\, df_i < \infty , \qquad i = 1, \ldots, m ; \quad j = 1, \ldots, l .$$

A randomized decision function $\eta(x)$, say, hereafter often called "test" for short, is defined as follows: $\eta(x) = [\eta_1(x), \eta_2(x), \ldots, \eta_l(x)]$ where

(a) $\eta(x)$ is defined for all x,

(b) $0 \leqq \eta_j(x), j = 1, \ldots, l,$

(c) $\sum_{j=1}^{l} \eta_j(x) = 1$ identically in x,

(d) $\eta_j(x)$ is measurable, $j = 1, \ldots, l$.

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[1] The extension to general abstract spaces is trivial and we forego it. This entire paper could be given an abstract formulation without the least mathematical difficulty.