ASYMPTOTICALLY SUBMINIMAX SOLUTIONS OF COMPOUND STATISTICAL DECISION PROBLEMS

HERBERT ROBBINS
UNIVERSITY OF NORTH CAROLINA

1. Summary

When statistical decision problems of the same type are considered in large groups the minimax solution may not be the "best," since there may exist solutions which are "asymptotically subminimax." This is shown in detail for a classical problem in the theory of testing hypotheses.

2. Introduction

Consider the following *simple statistical decision problem*. The random variable x is normally distributed with variance 1 and mean θ, where θ is known to have one of the two values ± 1. It is required to decide, on the basis of a single observation on x, whether the true value of θ is 1 or -1, in such a way as to minimize the probability of error.

For any decision rule R the probability of error will depend on the true value of θ. Let

(1) $$\eta(R) = P[\text{error} \mid R, \theta = -1], \quad \delta(R) = P[\text{error} \mid R, \theta = 1].$$

By a suitable choice of R we can give to $\eta(R)$ any desired value between 0 and 1; unfortunately, if R is chosen so that $\eta(R)$ is near 0 then $\delta(R)$ will be near 1, and in this circumstance lies the problem.

For any constant c let R_c be the decision rule which asserts "$\theta = \operatorname{sgn}(x - c)$"; thus in using R_c we assert "$\theta = 1$" if $x > c$ and "$\theta = -1$" if $x < c$. Then

(2) $$\begin{aligned} \eta(R_c) &= \int_c^{\infty} f(x+1)\, dx = F(-1-c), \\ \delta(R_c) &= \int_{-\infty}^{c} f(x-1)\, dx = F(-1+c), \end{aligned}$$

where we have set

(3) $$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \qquad F(x) = \int_{-\infty}^{x} f(y)\, dy = 1 - F(-x).$$

It is clear from (2) that

(4) for any number η between 0 and 1 there exists a number $c = c(\eta)$ such that $\eta(R_c) = \eta$.