

# A GENERALIZED $T$ TEST AND MEASURE OF MULTIVARIATE DISPERSION

HAROLD HOTELLING

UNIVERSITY OF NORTH CAROLINA

## CONTENTS

	Page
1. Introduction . . . . .	23
2. Generalized Student ratios and discriminant functions . . . . .	24
3. Figurative distance and coefficient of racial likeness . . . . .	26
4. Approximate $T$ distribution for large samples . . . . .	27
5. Ballistic and quality control applications . . . . .	28
6. Distributions of the $T$ statistics in two dimensions . . . . .	32
7. The general degree of dispersion of a multivariate normal distribution . . . . .	36
8. Efficiency of mean square circular error . . . . .	40

## 1. Introduction

A set of distributions advanced here generalizes to more than one dimension the analysis of variance of R. A. Fisher, which is itself a generalization including several simpler tests as special cases. The first of these, a generalization of the Student distribution to  $p$  dimensions, was introduced in 1931, and provides a test of significance for discriminant functions. The others developed initially in connection with a wartime problem of air testing sample bombsights, which led to a reconsideration of the whole problem of measuring ballistic dispersion. The statistical tests found efficient for these purposes were then seen to be suitable for a variety of industrial quality control and sampling inspection problems, particularly those involving multiple measurements, as on complicated and expensive assemblies and performance tests. These statistical methods are also related to certain procedures developed in India in connection with anthropometric investigations.

Of the bombsight report [9] the larger and less mathematical part has now been published [11]; this published work gives details regarding certain applications of the sampling distributions for which the fundamental mathematics, originally written as appendices to the bombsight report, appears publicly for the first time in the last three sections of this paper.

The multivariate generalization of the analysis of variance is not uniquely confined to the distributions here considered. In passing from one to a plurality of dimensions the variety of null hypotheses of interest is greatly multiplied, and the

Completion of this investigation was aided by the Office of Naval Research under Project NR-042 031 for research in multivariate analysis at Chapel Hill. Preliminary versions were embodied in two appendices of a report [9] prepared during the war, in a paper presented at a meeting of the Institute of Mathematical Statistics at Atlantic City, January 25, 1947, and in an abstract [10].