# ASYMPTOTIC MINIMAX SOLUTIONS OF SEQUENTIAL POINT ESTIMATION PROBLEMS

A. WALD
COLUMBIA UNIVERSITY

## 1. Introduction

Exact minimax solutions for sequential point estimation problems are, in general, very difficult to obtain. As far as the author is aware, such solutions are known, at present, only in two special cases: (1) in estimating the mean of a normal distribution with known variance (see Wolfowitz [1]) and (2) in estimating the mean of a rectangular distribution with unit range (see Wald [2]). The solution in the first case coincides with the classical nonsequential one, while the solution in the second case is truly sequential.

In this note, we shall derive an asymptotic minimax solution for a general class of point estimation problems. The point estimation problem considered here may be stated as follows: Let $\{X_i\}$ ($i = 1, 2, \ldots$, ad inf.) be a sequence of independently and identically distributed chance variables. Let $F(u|\theta)$ be the common distribution function involving an unknown parameter $\theta$, that is, $Pr\{X < u\} = F(u|\theta)$. We shall assume that $F(u|\theta)$ admits a density function $f(u|\theta)$. A sequential point estimation procedure $T$ can be defined in terms of two sequences of functions $\{\varphi(x_1, \ldots, x_m)\}$ and $\{t(x_1, \ldots, x_m)\}$ ($m = 1, 2, \ldots$, ad inf.) where $\varphi(x_1, \ldots, x_m)$ can take only the values 0 and 1. The estimation procedure is then given as follows: Let $x_i$ denote the observed value of $X_i$. We continue taking observations as long as $\varphi(x_1, \ldots, x_m) = 0$. At the first time when $\varphi(x_1, \ldots, x_m) = 1$, we stop experimentation and estimate the unknown parameter value by $t(x_1, \ldots, x_m)$. We shall assume that the cost of experimentation is proportional with the number of observations. Let $c$ denote the cost of a single observation and let the loss due to estimating the true parameter value $\theta$ by $t$ be given by $(t - \theta)^2$.

Let $\nu(\theta, T)$ denote the expected number of observations when $\theta$ is the true parameter value and the estimation procedure $T$ is adopted. Furthermore, let $\rho(\theta, T)$ be the expected value of $(t - \theta)^2$ when $\theta$ is true and $T$ is adopted. This expected value is given by

$$(1.1)\quad \rho(\theta, T) = \sum_{m=1}^{\infty} \int_{R_m} [t(x_1, \ldots, x_m) - \theta]^2 f(x_1|\theta) \ldots \times f(x_m|\theta)\, dx_1 \ldots dx_m$$

where $R_m$ is the totality of all sample points $(x_1, \ldots, x_m)$ for which $\varphi_i(x_1, \ldots, x_i) = 0$

This research was done under contract with the Office of Naval Research.