EXTENSION OF THE ROMANOVSKY-BARTLETT-SCHEFFÉ TEST

EDWARD W. BARANKIN

UNIVERSITY OF CALIFORNIA, BERKELEY

1. Introduction

We are concerned here with finding a suitable test for the equidependence of the means of two normal populations on respective linear regression variables (which may be identical) when no information is at hand regarding the two variances involved. More precisely, let x be a random variable, normally distributed with mean $h_1 + k_1 \xi$ and variance σ_1^2. Here h_1 and k_1 are constants, and the mean thus depends linearly on the single sure variable ξ. The variance σ_1^2 is independent of ξ. Similarly, let y be a normal random variable, with mean $h_2 + k_2\eta$ and variance σ_2^2. Here, likewise, h_2 and k_2 are constants, η is a sure variable and σ_2^2 is independent of η. Under the set of alternatives

$$\begin{cases} -\infty < h_1, h_2, k_1, k_2 < \infty \\ 0 < \sigma_1^2, \sigma_2^2 \end{cases}$$

we seek an exact, unbiased test for the hypothesis

$$H_0: \quad k_1 = k_2.$$

The test will have reference to a sample $(x_1, x_2, \cdots, x_m) \sim (\xi_1, \xi_2, \cdots, \xi_m)$ out of the first population, and a sample $(y_1, y_2, \cdots, y_n) \sim (\eta_1, \eta_2, \cdots, \eta_n)$ out of the second. The notation here is meant to imply that the random values (or variables) x_i, y_j are observed when ξ and η have the values ξ_i, η_j, respectively. With no loss of generality, we may assume $n \geqq m$.

In this problem [(P), for brevity], as in that of the comparison of constant means of two normal populations with unknown variances, the question of a *best* exact test must for the present go unanswered, for want of sufficiently powerful methods of determining all similar regions. V. Romanovsky, M. S. Bartlett, and H. Scheffé, in obtaining solutions of the latter problem, have brought to bear a specialized procedure based on "Student's" t-test, and with it have produced exact, unbiased tests in that case. The procedure can be applied to (P) as well (and to a large class of problems, in fact), to yield a test of the same character; and one which, like those of Romanovsky, Bartlett, and Scheffé for constant means, has the advantages of having a simple criterion and requiring only the use of t-tables.

The first steps in fashioning and applying the procedure were taken by Romanovsky [**3**][1] in 1928. It appears, however, that Romanovsky's paper was entirely overlooked and the method was later rediscovered by Bartlett in

[1] Boldface numbers in brackets refer to references at the end of the paper (p. 449).