# ON THE THEORY OF STOCHASTIC PROCESSES, WITH PARTICULAR REFERENCE TO APPLICATIONS

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## 1. Introduction

Since Kolmogoroff's famous paper of 1931, "On Analytical Methods in the Theory of Probability," the theory of stochastic processes has been developed and it has been shown that it can successfully be applied to practical problems and used to describe empirical phenomena. However, the theory is new and the most appropriate mathematical techniques have yet to be discovered. It is therefore reasonable to expect that the usefulness of the theory will increase when more pertinent mathematical problems are solved. On the other hand, these new problems are of interest also in pure analysis beyond the theory of stochastic processes. In the past, pure mathematics has always derived great benefits from the interplay with physical theories, and many parts of purest mathematics owe their origin to physical problems. Now we shall see that our theory leads to integrodifferential equations of a type never studied before: they contain as simplest special cases a surprisingly great variety of familiar and unfamiliar functional equations (sec. 7). It seems probable that our methods of deriving solutions and adjoint equations could be utilized also for many functional equations without probabilistic meaning.[1] Another example of a problem of general interest on which we shall touch briefly (sec. 10) is connected with the fact that an empirical phenomenon can often be described in several ways, say by a system of infinitely many ordinary differential equations or by a partial differential equation of the diffusion type. This seems to indicate connections which have yet to be explored.

As for practical usefulness, it should be borne in mind that for a mathematical theory to be applicable it is by no means necessary that it be able to provide accurate models of observed phenomena. Very often in applications the constructive role of mathematical theories is less important than the economy of thought and experimentation resulting from the ease with which qualitatively reasonable working hypotheses can be eliminated by mathematical arguments. Perhaps even more important is the constant interpretation of observations in the light of theory and of theory in the light of observations; in this way mathematical theory can become an indispensable guide not only to a better understanding, but even to a proper formulation of scientific

[1] The integrodifferential equations mentioned in the text contain as a special case, among others, infinite systems of ordinary differential equations, which will be studied in section 4. The theory of these ordinary differential equations has been generalized also to cases which are void of probabilistic meaning [cf. Arley and Borchsenius (1945)].