THE LIMITING DISTRIBUTION OF FUNCTIONS OF SAMPLE MEANS AND APPLICATION TO TESTING HYPOTHESES

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Introduction

In 1935 J. L. Doob published a paper **[2]**[1] in which he derived the limiting distribution of a function of four sample means from one homogeneous sample. This work is susceptible to an easy generalization and supplies a powerful weapon with which to find the limiting distribution of a vast number of statistics. But since publication its importance seems to have been overlooked. A generalization of Doob's theorem to any number of sample means was given by the author **[7]**.

In the first part of this paper two theorems are proved which embody a further generalization of Doob's result to the case of several samples of different sizes, and numerous examples are given to illustrate their wide applicability.

These examples are confined to the limiting distributions of given statistics, but in the second part a much more important constructive application is made. Two hypotheses of a general character, concerning one sample and several samples respectively, are formulated, and a systematic method of constructing a test function for each hypothesis included in the two general ones is given. The construction is done in such a manner that, as a consequence of the results obtained in the first part, (i) the test function has for its limiting distribution the χ^2 distribution with a known degree of freedom when the hypothesis tested is true, and (ii) the power of the test tends in general to unity as its limit. Special hypotheses and their large sample tests are treated as examples in the second part of the paper.

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The limiting distribution of functions of sample means

1. *The mathematical model of* k *samples.*—Let there be given k random vectors of m components each,

$$\mathbf{u}_a = [U_{1a}, U_{2a}, \cdots, U_{ma}], \qquad a = 1, \cdots, k, \tag{1}$$

possessing finite second moments. Let

$$E(U_{ia}) = \mu_{ia}, \qquad E(U_{ia}U_{ja}) - \mu_{ia}\mu_{ja} = \eta_{ija}. \tag{2}$$

[1] Boldface numbers in brackets refer to references at the end of the paper (p. 402).