AN EXTENSION OF AN ALGORITHM OF HOTELLING

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1. Introduction

The algorithm in question is one for the rapid approximate computation of the inverse to a linear operator. It has become known to statisticians and others through the work of Hotelling [1],[1] who used it for inverting finite matrices—linear operators in finite-dimensional vector spaces—and found a bound for the error. Somewhat earlier, Ostrowski [4] had proposed its use in a rather general class of problems, with special emphasis on integral equations of second kind and Volterra type. He did not give a bound for the error. More recently Rademacher [5] has applied the same idea to calculating Laurent expansions of algebraic functions.

The object of this paper is to show how the error can be limited in a rather general problem, and to suggest specific procedures for applying it to linear integral equations of second kind and Fredholm type.

2. Vector spaces

For this purpose it may be useful to state some facts about normed linear vector spaces. Suppose L is such a space (which will for definiteness be assumed real): that is, if x_1 and x_2 are vectors in L, so is $ax_1 + bx_2$, where a and b are real numbers; with every x in L is associated a non-negative real number $\|x\|$, the norm of x; $\|x\| > 0$ unless x is the zero-element of L, whose norm is zero. Furthermore, $\|x_1 + x_2\| \leqq \|x_1\| + \|x_2\|$, and $\|ax\| = |a| \cdot \|x\|$.

We consider an operator K which transforms any element x of L into an element Kx of L; the special operator I is that which carries each x of L into itself. We suppose K to be additive, homogeneous, and continuous (i.e., linear); there then exists a constant

$$M(K) = \underset{x \in L}{\text{l.u.b.}} \frac{\|Kx\|}{\|x\|}, \tag{2.1}$$

called the (upper) bound, or norm, of K. Clearly, $M(I) = 1$. If

$$M(K) < 1, \tag{2.2}$$

the equation

$$x - Kx \equiv (I - K)x = y \tag{2.3}$$

[1] Boldface numbers in brackets refer to references at the end of the paper (see p. 358).