REMARKS ON COMPUTING THE PROBABILITY INTEGRAL IN ONE AND TWO DIMENSIONS

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Introduction

In the first part of the present paper the probability integral in one dimension is considered. This first part may be regarded as an illustration of the principle that "no problem whatever is solved completely." In fact, the problem of computing the total area under the Gaussian curve was solved by Laplace or, even before him, under a slightly different form, by Euler, and this solution has been presented since under various forms. In the present paper there are offered two different solutions of the same problem, and an inequality, derived from Laplace's solution, all of which seem to be new and are certainly very little known.

In the second part of the paper the probability integral in two dimensions is considered. In this part, which had its origin in a practical problem, formulas and inequalities which appear to be useful in computing volumes under the normal probability surface are presented.

We use the following notation:

$$g(x) = (2\pi)^{-1/2} e^{-x^2/2}, \tag{1}$$

$$G(x) = \int_0^x g(t)dt, \tag{2}$$

$$\begin{aligned} L &= L(a,a';b,b';r) \\ &= \int_a^b \int_{a'}^{b'} [2\pi(1 - r^2)]^{-1/2} g\{[(x^2 - 2rxx' + x'^2)/(1 - r^2)]^{1/2}\} dx'dx, \end{aligned} \tag{3}$$

$$M = M(h,k;r) = L(h,k; +\infty, +\infty; r). \tag{4}$$

The symbols $g(x)$ and $G(x)$ should remind us of "Gauss." The limits a,b in L correspond to x, and a',b' to x'. The quantity M is represented by an integral of the type

$$\int_h^\infty \int_k^\infty .$$

I. The Probability Integral in One Dimension

1. An inequality

We try to see something new in the most usual method of evaluating the total area under the Gaussian curve. Following that method, we consider