A POSTULATIONAL CHARACTERIZATION OF STATISTICS

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1. Introduction

We shall present a system of postulates which will be shown to constitute an adequate basis for statistics. The fundamental elements in the system are variates, and we assume that these variates behave very much like numbers in ordinary algebra. In particular

$$x + y, \qquad x \cdot y, \qquad -x$$

are variates if x and y are variates. An observation of a variate is a number, and an observation of the variate $x + y$ is the sum of the corresponding observations made on the variates x and y. Similarly for the product and the negative. Since some of the observations of a variate may be zero, we should not expect division to be defined. We should, however, expect the presence of a zero variate 0 and a unity 1. Thus our system is just what algebraists describe as a commutative ring with a unity.

A fortuitous event is a special case of a variate. An observation of such a variate is either a success (represented by a 1) or a failure (represented by a 0). Since

$$1 \cdot 1 = 1 \quad \text{and} \quad 0 \cdot 0 = 0,$$

we might expect a fortuitous event x to be an idempotent variate, that is, one such that

$$x \cdot x = x.$$

The product $x \cdot y$ of two events can be interpreted as the conjunction of the events, that is, the event "x and y." Thus an observation of $x \cdot y$ is a 1 (success) if and only if the corresponding observations of the factors are both 1's (successes).

The disjunction of x and y is symbolized by $x \vee y$ (read "x or y") and is defined by the equation

$$x \vee y = x + y - x \cdot y.$$

An observation of $x \vee y$ succeeds if and only if the corresponding observation of at least one member of the disjunction succeeds.

The unity 1 satisfies the equations

$$1 \cdot x = x \cdot 1 = x$$