# PHILOSOPHICAL FOUNDATIONS OF PROBABILITY

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## I

In sections I–V we deal with the formal structure of probability; in sections VI–XI we investigate the meaning and the assertability of probability statements.[1]

The concept of probability refers to a relation. If we cast a die, the probability of a certain face is 1/6; the condition introduced by "if" is necessary for this instance of a probability relation as well as for all others. When the condition is omitted the statement must be regarded as elliptic; such omission is possible if it is obvious from the context what condition is understood. We therefore regard probability as having the logical form of an implication, which we call the *probability implication.*

This implication, however, holds not between individuals but between classes. Thus the phrase "cast the die" defines a class $A$ of events, and similarly the phrase "face 6 turns up" defines a class $B$ of events. The class $A$ is called the *reference class,* the class $B$ is named the *attribute class.* Furthermore, the events $x_i$ and $y_i$ belonging respectively to these classes are regarded as given in a certain order and in such a way that a one-to-one correspondence between the elements of the sequences is known, which we express by the use of the same subscript "$i$". Since the probability statement refers to all events belonging to the classes $A$ and $B$, it can be written in the form of an all-statement:

$$(i)\ \ (x_i \epsilon A \underset{p}{\ni} y_i \epsilon B) \tag{1}$$

The symbol "$(i)$" is the all-operator of logistics; the symbol "$\epsilon$", as usual, denotes the relation of class membership. The real number $p$ is the degree of probability.

Instead of the *implicational notation* presented in (1) it is convenient to introduce a *mathematical notation,* or *functor notation.* We write

$$P(A,B) = p \tag{2}$$

The symbol "$P(\ \ )$" is a functor, meaning "the probability of". Expression (2) has the same meaning as (1) and can be regarded as an abbreviation in which the reference to the sequences $x_i$ and $y_i$ is not expressed. We read (2) in the form "the probability from $A$ to $B$ is $p$".

[1] For a detailed account of the following ideas we refer the reader to the author's *Wahrscheinlichkeitslehre* (Leiden, 1935), A. W. Sijthoff. A summary in the French language was published under the title "Les fondements logiques du calcul des probabilités," *Annales de l'Institut Henri Poincaré,* t. VII, fasc. v (Paris, 1937), pp. 267–348. The general ideas of secs. VI–XI are presented in chap. v of the author's *Experience and Prediction* (Chicago, 1938).