## LOCAL TRUTH

"a Grothendieck topology appears most naturally as a modal operator, of the nature 'it is locally the case that'"

F. W. Lawvere

The notion of a topological bundle represents but one side of the coin of sheaf theory. The other involves the conception of a sheaf as a functor defined on the category of open sets in a topological space. Our aim now is to trace the development of ideas that leads from this notion, via Grothendieck's generalisation, to the notion of a "topology" on a category and its attendant sheaf concept, and from there to the first-order concept of a topology on a topos and the resultant axiomatic sheaf theory of Lawvere and Tierney. The chapter is basically a survey, and its intention is to direct the reader to the appropriate literature.

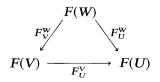
## 14.1. Stacks and sheaves

Let I be a topological space, with  $\Theta$  its set of open subsets.  $\Theta$  becomes a poset category under the set inclusion ordering, in which the arrows are just the inclusions  $U \hookrightarrow V$ .

A stack or pre-sheaf over I is a contravariant functor from  $\Theta$  to Set. Thus a stack F assigns to each open V a set F(V), and to each inclusion  $U \hookrightarrow V$  a function  $F_U^V: F(V) \to F(U)$  (note the contravariance – reversal of arrow direction), such that

(i)  $F_U^U = id_U$ , and

(ii) if  $U \subseteq V \subseteq W$ , then



commutes, i.e.  $F_U^W = F_U^V \circ F_V^W$ .