APPENDIX. SOME RESULTS FOR PLANAR GRAPHS.

In this appendix we prove several graph theoretical, or point-set topological results, in particular Propositions 2.1-2.3 and Corollary 2.2 which were already stated in Ch.2. The proofs require somewhat messy arguments, even though most of these results are quite intuitive. We base most of our proofs on the Jordan curve theorem (Newman,(1951), Theorem V. 10.2). Some more direct and more combinatorial proofs can very likely be given; see the approach of Whitney (1932, 1933). Especially Whitney (1933), Theorem 4, is closely related to Cor. 2.2., Prop. 2.2 and Prop. A.1, and has been used repeatedly in percolation theory.

Throughout this appendix $\mathcal{M}$ is a mosaic, $\mathcal{F}$ a subset of the collection of faces of $\mathcal{M}$ and $(\mathcal{G},\mathcal{G}^*)$ a matching pair based on $(\mathcal{M},\mathcal{F})$. These terms were defined in Sect. 2.2. $\mathcal{G}_{p\ell},\mathcal{G}^*_{p\ell}$ and $\mathcal{M}_{p\ell}$ will be the planar modifications as defined in Sect. 2.3. We fix an occupancy configuration ω on $\mathcal{M}$ and extend it as in (2.15), (2.16). W(v) and $W_{p\ell}(v)$ are the occupied cluster of v on $\mathcal{G}$ and $\mathcal{M}_{p\ell}$ (or $\mathcal{G}_{p\ell}$), respectively, in the configuration ω. ∂W, the boundary of W, is defined in Def. 2.8; $v\ \mathcal{G}\ w$ means that v and w are adjacent vertices on $\mathcal{G}$.

Proposition 2.1. Let $\partial W_{p\ell}(v)$ be the boundary of $W_{p\ell}(v)$ on $\mathcal{M}_{p\ell}$. If $W_{p\ell}(v)$ is non-empty and bounded and (2.3)-(2.5) hold with $\mathcal{G}$ replaced by $\mathcal{M}$, then there exists a vacant circuit $J_{p\ell}$ on $\mathcal{M}_{p\ell}$ surrounding $W_{p\ell}(v)$, and such that all vertices of $\mathcal{M}_{p\ell}$ on $J_{p\ell}$ belong to $\partial W_{p\ell}(v)$.

We owe the idea of the proof to follow to R. Durrett. We shall write $W_{p\ell}$ and $\partial W_{p\ell}$ instead of $W_{p\ell}(v)$ and $\partial W_{p\ell}(v)$. On various occasions we shall use the symbol for a path to denote the set of points which belong to some edge in the path. Thus in (A.2), the left hand side is the set of points which belong to π and to $W \cup \partial W_{p\ell}$. In (A.5) $\mathrm{int}(J) \setminus \tilde{\pi}$ is the set of points in int(J) which do not lie