11. RESISTANCE OF RANDOM ELECTRICAL NETWORKS.

11.1 Bounds for resistances of networks.

Many people have studied the electrical resistance of a network made up of random resistors. It was realized quite early that critical phenomena occur, and that there is a close relation with percolation theory, in special cases where the individual resistors can have infinite resistance (or zero resistance). We refer the reader to Kirkpatrick (1978) and Stauffer (1979) for a survey of much of this work. In these introductory paragraphs we shall assume that the reader knows what the resistance of a network is, but we shall come back to a description of resistance in Sect. 11.3.

A typical problem in which the relation with percolation is apparent is the following. Consider the graph $\mathbb{Z}^d$, with vertices the integral vectors in $\mathbb{R}^d$, and edges between two vertices v_1 and v_2 iff $|v_1-v_2| = 1$. Assume each edge of $\mathbb{Z}^d$ is a resistance of 1 ohm with probability p, and is removed with probability $q = 1-p$. As usual all edges are assumed independent of each other. Let $\mathcal{H}_n$ be the restriction of the resulting random network to the cube of size n, $B_n = [0,n]^d$. What is the behavior for large n of the resistance in $\mathcal{H}_n$ between the left and right face of B_n? More precisely let

$$A^0 = A_n^0 = \{v = (v(1),\dots,v(d)): v(1) = 0, 0 \le v(i) \le n, \; 2 \le i \le d\} \tag{11.1}$$

be the left face of B_n and

$$A^1 = A_n^1 = \{v = (v(1),\dots,v(d)): v(1) = n, 0 \le v(i) \le n, \; 2 \le i \le d\} \tag{11.2}$$

the right face. Form a new network from $\mathcal{H}_n$ by identifying as one vertex a_0 all vertices of $\mathbb{Z}^d$ in A^0, and by identifying all vertices of $\mathbb{Z}^d$ in A^1 as another vertex a_1. This means that we view all edges of $\mathcal{H}_n$ which run between the hyperplanes $x(1) = 0$ and $x(1) = 1$ as having the common endpoint a_0 in $x(1) = 0$. In "reality"