8. POWER ESTIMATES.

In this chapter we study the behavior of the percolation probability and the expected size of an occupied cluster in a one-paremeter problem. As defined in Ch. 3 this means that we consider probability measures for which

$$P_p\{v \text{ is occupied}\} = p$$

is the same for all vertices v of the studied graph $\mathcal{G}$, and the occupancies of all vertices are independent. We want to know the asymptotic behavior of

$$\theta(p) = \theta(p,z_0) = P_p\{\#W(z_0) = \infty\}$$

and of [1)]

$$E_p\{\#W(z_0);\ \#W(z_0) < \infty\}$$

as p approaches the critical probability p_H (see Sect. 3.4). By analogy with results in statistical mechanics, and on the basis of numerical evidence (see Stauffer (1979) and Essam (1980)) it is generally believed that

$$(8.1) \qquad \theta(p) \sim C_0(p-p_H)^{\beta}, \quad p \downarrow p_H,$$

$$(8.2) \qquad E_p\{\#W(z_0);\ \#W(z_0) < \infty\} \sim C_+(p-p_H)^{-\gamma+}, \quad P \downarrow p_H$$

and

$$(8.3) \qquad E_p\{\#W(z_0)\} \sim C_-(p_H-p)^{-\gamma-}, \quad p \uparrow p_H$$

for suitable constants C_0 $C_\pm$ and $0 < \beta, \gamma_\pm < \infty$. Similar power laws are conjectured for other quantities. It is also conjectured that the so-called critical exponents $\beta, \gamma_\pm$ do not depend (or

1) $E\{X;A\}$ stands for $E\{X I_A\}$, i.e., the integral of X over the set A.