## 5. BOUNDS FOR THE DISTRIBUTION OF # W .

The principal result of this chapter is that

(5.1) $$P_p\{\#W(v) \geq n\}$$

decreases exponentially in n, provided certain crossing probabilities are sufficiently small. This is almost the only theorem which works for a general periodic percolation problem in any dimension. No axes of symmetry are required, nor does the graph have to be one of a matching pair. When Theorem 5.1 is restricted to one-parameter problems, then it shows that (5.1) decreases exponentially for $p < p_T$ and that in general $p_T = p_S$ (see (3.62)-(3.65) for definition). In Sect. 5.2 we discuss lower bounds for

(5.2) $$P_p\{\#W(v) = n\}$$

when $p$ is so large that percolation occurs. In the one-parameter case this is the interval $p_H < p \leq 1$. It turns out that (5.2), and hence (5.1) does not decrease exponentially in this domain. We have no estimates for (5.1) for $p$-values at which

(5.3) $$E_p\{\#W(v)\} = \infty, \text{ but } \theta(p,v) = P_p\{\#W(v) = \infty\} = 0 ,$$

except in the special cases of $\mathcal{G}_0$ and $\mathcal{G}_1$ (see Theorem 8.2). Of course if Theorem 3.1 and Cor. 3.1 apply then (5.3) can happen only on the critical surface, and one may conjecture that in general the set of $p$-values at which (5.3) holds has an empty interior. In one-parameter problems this amounts to the conjecture that $p_T = p_H$ in all periodic percolation problems. If one goes still further one might conjecture that (5.1) decreases only as a power of $n$ whenever (5.3) holds. For bond- or site-percolation on $\mathbb{Z}^2$, Theorem 8.2 indeed gives a lower bound of the form $n^{-\gamma}$ for (5.1) at $p = p_H$ .

In Sect. 5.3 we discuss a result of Russo (1981) which is more or