4. INCREASING EVENTS.

This chapter contains the well known FKG inequality and a formula of Russo's for the derivative of $P_p\{E\}$ with respect to p for an increasing event E. No periodicity assumptions are necessary in this chapter, so that we shall take as our probability space the triple $(\Omega_{\mathcal{V}}, \mathcal{B}_{\mathcal{V}}, P_{\mathcal{V}})$ as defined in Sect. 3.1. $E_{\mathcal{V}}$ will denote expectation with respect to $P_{\mathcal{V}}$.

Def. 1 A $\mathcal{B}_{\mathcal{V}}$ -measurable function $f:\Omega_{\mathcal{V}} \to \mathbb{R}$ is called increasing (decreasing) if it is[1] increasing (decreasing) in each $\omega(v)$, $v \in \mathcal{V}$. An event $E \in \mathcal{B}_{\mathcal{V}}$ is called increasing (decreasing) if its indicator function is increasing (decreasing).

Examples

(i) $\{\#W(v)\}$ is an increasing function, since making more sites occupied can only increase $W(v)$.

(ii) $E_1 = \{\#W(v) = \infty\}$ for fixed v is an increasing event; if E_1 occurs in the configuration ω', and every site which is occupied in ω' is also occupied in ω'' - and possibly more sites are occupied in ω'' - then E_1 also occurs in configuration ω''.

(iii) $E_2 = \{\exists$ an occupied path on $\mathcal{G}$ from v_1 to $v_2\}$ for fixed vertices v_1 and v_2 is increasing for the same reasons as E_1 in ex. (ii).

(iv) The most important example of an increasing event for our purposes is the existence of an occupied crosscut of a certain Jordan domain in $\mathbb{R}^2$. More precisely we shall be interested in pair of matching graphs $(\mathcal{G},\mathcal{G}^*)$ in $\mathbb{R}^2$ based on $(\mathcal{M},\mathcal{F})$, $\mathcal{G}_{p\ell}$, $\mathcal{G}^*_{p\ell}$ and $\mathcal{M}_{p\ell}$ will be the planar modifications of $\mathcal{G}$, $\mathcal{G}^*$ and $\mathcal{M}$ respectively (see Sect. 2.2 and 2.3) . Let J be a Jordan curve on $\mathcal{M}_{p\ell}$

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1) We use "increasing" and "strictly increasing" instead of "non-decreasing" and "increasing".